MATH 2050C Lecture on 3/25/2020

* Important Announcement about Midterm *

- available on Apr 15, 2020 (Wed) at 8:30 am via BLACKBOARD
- submission deadline: Apr 16, 2020 (Thur) at 8:30 am
- Covers Ch. 2-4 (except binary/decimal representations, liminf/limsup, 3.6, 3.7, 4.3).

Common Mistake in HW: When ruling out the possibility $x := \lim(x_n) = 0$,

Some of you wrote " since $x_n > 0 \ \forall n \in \mathbb{N}$, we have $x > 0$ " NOT TRUE (e.g. $x_n = \frac{1}{n}$)

You need to show " $\exists\, c > 0$ s.t. $x_n \geq c > 0 \ \forall n \in \mathbb{N}$ ", then " $x \geq c > 0$ ".

Sequential Criteria: Let $f : A \subseteq \mathbb{R} \to \mathbb{R}$, $c \in \mathbb{R}$ be a cluster pt. of A.

$$\lim_{x \to c} f(x) = L \iff \forall \text{ seq. } (x_n) \text{ in } A \text{ satisfying } (*) \begin{cases} x_n \neq c & \forall n \in \mathbb{N} \\ \lim(x_n) = c \end{cases}$$

(ε-δ def$^{\underline{n}}$)

we have $\lim (f(x_n)) = L$

(ε-K def$^{\underline{n}}$)

This has 2 important consequences: (next time)

(i) thm / facts about limit of seq. carries over to limit of functions

(ii) not easy to prove existence of $\lim_{x \to c} f(x)$. BuT it is very helpful to prove non-existence of $\lim_{x \to c} f(x)$.

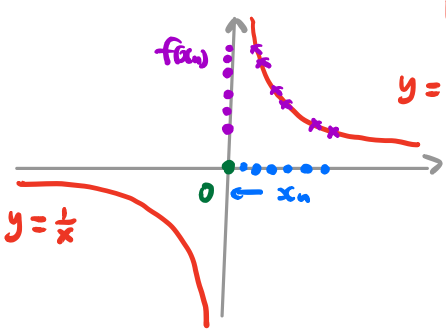
Divergence Criteria:

(I) f does NOT converge to L as $x \to c$ $\iff$ $\exists$ seq. (x_n) in A satisfying $(*)$ BuT $(f(x_n))$ does NOT converge to L

(II) f diverges as $x \to c$ (i.e. f does NOT converge to L as $x \to c$ $\forall L \in \mathbb{R}$) $\iff$ $\exists$ seq. (x_n) in A satisfying $(*)$ BuT $(f(x_n))$ is divergent

Example 1: $\lim_{x \to 0} \frac{1}{x}$ does NOT exist.

$f(x) = \begin{cases} 1/x & \text{if } x \neq 0 \\ \text{undefined} & \text{at } x = 0 \end{cases}$

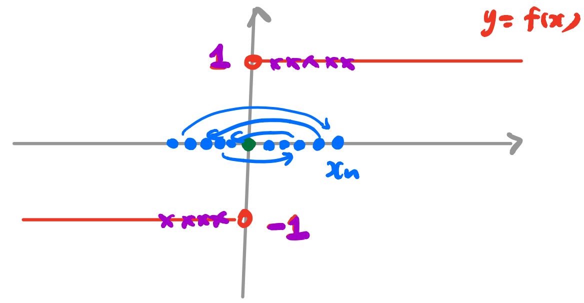


Consider $(x_n) = (\frac{1}{n})$ satisfy $(*)$

BuT $(f(x_n)) = (n)$ is divergent

By (II), we are done.

Example 2: $\lim_{x \to 0} \frac{x}{|x|}$ does NOT exist.



Consider $(x_n) = \left(\frac{(-1)^n}{n}\right)$ satisfying $(*)$

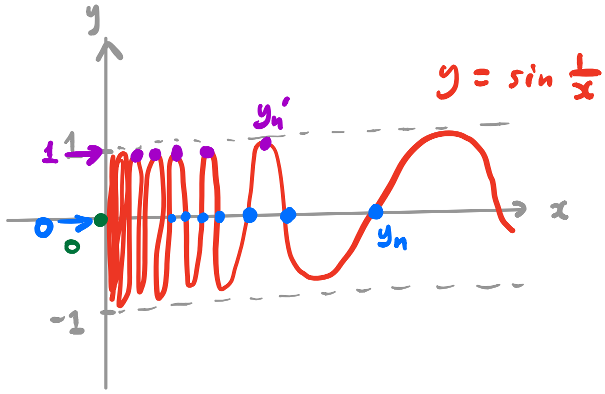
BuT $(f(x_n)) = ((-1)^n)$ is divergent.

By (II), we are done.

Example 3: $\lim_{x \to 0} \sin\frac{1}{x}$ does NOT exist.

$f(x) = \sin\frac{1}{x}$

well-defined except at $x=0$



topologist's sine curve

Take $(y_n) := \left(\frac{1}{n\pi}\right) \to 0$, then $(f(y_n)) = \left(\sin\frac{1}{1/n\pi}\right) = (0) \to 0$

Take $(y_n') := \left(\frac{1}{\frac{\pi}{2}+2n\pi}\right) \to 0$, then $(f(y_n')) = \left(\sin\left(\frac{\pi}{2}+2n\pi\right)\right) = (1) \to 1$

Choose (x_n) to alternative between these 2 seq.:

$$(x_n) = (y_1, y_1', y_2, y_2', y_3, y_3', \dots) \xrightarrow{\text{Why?}} 0$$

AND: $(f(x_n)) = (0, 1, 0, 1, 0, 1, \dots)$ is divergent

By (II), we are DONE.

Remark: Example 3 gives a way to prove Divergence Criteria (II). (Pf: Ex.)